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Fusion channel of $pd\mu$ charge-symmetric ion including photons

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Abstract

The charge-symmetric pseudo nucleus $pd\mu$ can be formed in the cascade process in the muon catalyzed fusion. The nuclear fusion of the $pd\mu$ molecular ion can be considered in the photon field. For the spin states of the $pd\mu$ system ($L = 0$), the radiative fusion rates are calculated employing a new spatial wavefunction. The method takes into account the Coulomb interactions for the calculation of the molecular wavefunction. The related pd astrophysical factors are used, essentially extracted from experimental determinations.

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1. Introduction

The different results of calculations of fusion rates in the $pd\mu$ molecule reflect different approximations in the solution to the Schrödinger equation for three particles. The main uncertainty is associated with the results at small distances. When the adiabatic expansion is used; the important problem of convergence of the expansion at small distances is usually ignored. Such problems vanish if the direct solution of the Faddeev equations in the configuration space is performed [1–3]. More accurate approaches treat the nuclear force dynamically by directly incorporating it as a complex potential in the three-body Hamiltonian [4, 5], or as complex boundary conditions at the nuclear surface [6, 7]. These elaborate approaches agree rather well with each other, as well as with the simple factorization approach. All of the mentioned methods include a large volume of calculations. We consider the molecular and nuclear reactions of $pd\mu$ using a simple wavefunction to $pd\mu$. The $pd\mu$ molecule is a good example of using μ CF as a probe of nuclear reactions in few body systems [8, 9]. The radiative and non-radiative fusions can take place in this ion. The $pd\mu$ is one of the molecular ions which are not resonantly formed [10]. The Auger process dominates for its molecular states with binding energy larger than the ionization potential. Therefore, the study of the mentioned ion could be of interest in the Ramsauer–Townsend (RT) effect [10] in the

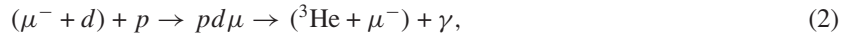
$\mu d + p$ scattering. The elastic scattering cross section has a deep minimum corresponding to zero of the s-wave scattering amplitude. As a result, hydrogen becomes nearly transparent for the muonic atom μd in the corresponding energy range. This effect is used in the TRIUMF experiment [11] where the μd atom is produced following μ transfer from muonic protium in a layer of solid hydrogen with a small fraction of deuterium. The emitted muonic atoms are used for measurements of the energy dependence of various reactions by the time-of-flight method. This paper is organized as follows. In section 2 we describe the theoretical framework of fusion calculations of the $pd\mu$ molecule, accounting for both the Coulomb potential and the nuclear factors. We develop the formulae including a new wavefunction for the estimation of the fusion rate in the photon field. In section 3, we summarize the paper and mention the future perspectives of this study as conclusions.

2. Transition in the μ -catalyzed pd fusion

The fusion via the nuclear reaction



was observed by Alvarez *et al* in the Berkeley laboratory [12]. This internal conversion competes with the more common radiative capture:



where the final muon is most likely to reside in the first atomic level of the residual He atom [13]. For the nuclear fusion process, the initial proton with spin 1/2 and deuteron with spin 1 can reside in the states with 3/2 (quartet) or 1/2 (doublet). The $M1$ (doublet) and (quartet) radiative capture transitions in the p - d system provide a critical testing ground for modern three-body calculations. Some of previous measurements of these transitions have been obtained from a precision study [9] of the $pd\mu$ fusion cycle. In that experiment, the Wolfenstein–Gerstein effect [14] was employed to vary the relative amounts of $S = 1/2$ and $S = 3/2$ nuclear spins in the $pd\mu$ molecule prior to fusion. The results indicated a doublet fusion rate of $0.35(2) \times 10^6 \text{ s}^{-1}$ and a quartet fusion rate of $0.11(1) \times 10^6 \text{ s}^{-1}$. Shortly thereafter, the transition rates for the $M1$ radiative capture in both the quartet and doublet initial states were obtained in an *ab initio* three-body calculation [15]. This calculation was performed with realistic NN interactions, including the three-body and Coulomb interactions, and, especially important for the case of the $M1$ transitions in ${}^3\text{He}$, it used explicit meson-exchange currents which had been parameterized to fit the thermal n - d capture cross section. The results of this calculation were in excellent agreement with the above experimental fusion rates. Furthermore, it was found that while the quartet capture is largely given by the impulse approximation, the doublet capture had very large exchange-current contributions. The result of that work resolved the so-called anomaly in the Wolfenstein–Gerstein effect, which was based on an erroneous no-quartet theorem [16]. The present work reports a new determination of the radiative fusion rate of $pd\mu$ in the muon catalyzed fusion, μCF , using the quartet and doublet experimental astrophysical factor (s-wave). The physical behavior at low energies is expected in the μCF . The $pd\mu$ molecule works as a partial-wave filter: the fusion always takes place in the ro-vibrational ground state of the μpd (as soon as the molecule is formed, it de-excites to the state $(J, \nu) = (0, 0)$ much faster than the fusion occurs), so that the pd system has the relative angular momentum $L = 0$ (s-wave). The radiative fusion rates depend on the total nuclear spin $\vec{s}_{pd} = \vec{s}_p + \vec{s}_d$. The radiative fusion rates including the quartet and doublet rates $\lambda_{3/2}^\gamma, \lambda_{1/2}^\gamma$ are given by the formulae [15]:

$$\lambda_{3/2}^\gamma = \frac{S_{3/2}(0)}{\pi \alpha c m_3} \rho_0, \quad \lambda_{1/2}^\gamma = \frac{S_{1/2}(0)}{\pi \alpha c m_3} \rho_0, \quad \lambda^\gamma = \frac{1}{3} \lambda_{1/2}^\gamma + \frac{2}{3} \lambda_{3/2}^\gamma = \frac{S(0)}{\pi \alpha c m_3} \rho_0 \quad (3)$$

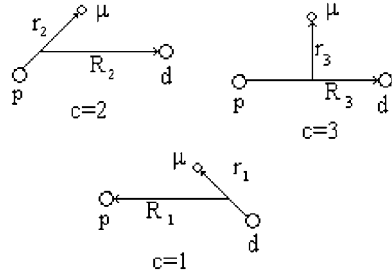


Figure 1. The three-channel configurations 1, 2 and 3.

where m_3 is the reduced mass of the two-body system of pd . $S(0)$ denotes (s-wave) the astrophysical factor for p - d capture at zero energy, which includes the two terms as

$$S(0) = (S_{1/2} + 2S_{3/2})/3. \quad (4)$$

These terms in laboratory experiments are given by

$$S_{1/2}(0) = \frac{2\pi}{3} \alpha^2 m_3 c^2 \left[\frac{\omega}{\hbar c} \right]^3 \left[\frac{\hbar}{2mc} \right]^2 \left| \left\langle \psi_{\text{He}}^{s_n=\frac{1}{2}} \left| M1 \left| \psi_{pd}^{L=0, s_{pd}=\frac{1}{2}} \right. \right. \right\rangle \right|^2, \quad (5)$$

$$S_{3/2}(0) = \frac{\pi}{3} \alpha^2 m_3 c^2 \left[\frac{\omega}{\hbar c} \right]^3 \left[\frac{\hbar}{2mc} \right]^2 \left| \left\langle \psi_{\text{He}}^{s_n=\frac{1}{2}} \left| M1 \left| \psi_{pd}^{L=0, s_{pd}=\frac{3}{2}} \right. \right. \right\rangle \right|^2.$$

The relations above show that the radiative capture can be performed in $M1$ transitions. ω is the frequency of photon γ , m being the mass of a nucleon (p or n). $\psi_{\text{He}}^{s_n=\frac{1}{2}}$ and $\psi_{pd}^{L=0, s_{pd}}$ are the wavefunctions of ${}^3\text{He}$ nucleus and p - d state, respectively. The notation ρ_0 is used for the molecular probability density of fusion in zero nuclear distance:

$$\rho_0 = \int |\psi_I(\vec{r}_\mu, \vec{r}_{pd} = 0)|^2 d\vec{r}_\mu d\Omega_{\vec{r}_{pd}} \quad (6)$$

where ψ_I is the spatial wavefunction of $pd\mu$. This wavefunction is needed for the rate calculations. We study the spatial situation of the $pd\mu$ molecule in the three coupled channels as follows. $(d\mu)$ - p , $(p\mu)$ - d and (pd) - μ are referred to channels $c = 1, 2$ and 3 , respectively, and their Jacobian coordinates (\vec{r}_c, \vec{R}_c) are defined as in figure 1. We introduced the reduced masses (m_c, M_c) which are associated with (\vec{r}_c, \vec{R}_c) . The mentioned masses are given by $m_1^{-1} = m_d^{-1} + m_\mu^{-1}$, $M_1^{-1} = m_p^{-1} + (m_d + m_\mu)^{-1}$, $m_2^{-1} = m_p^{-1} + m_\mu^{-1}$, $M_2^{-1} = m_d^{-1} + (m_p + m_\mu)^{-1}$, $m_3^{-1} = m_\mu^{-1} + (m_p + m_d)^{-1}$ and $M_3^{-1} = m_p^{-1} + m_d^{-1}$. The spatial wavefunction of $pd\mu$ and the Hamiltonian H are introduced respectively as

$$\psi_I = \sum_{c=1}^3 \Psi_{JM}^{(c)}(\vec{r}_c, \vec{R}_c) Y_{J,M}(\hat{R}_c) = \frac{1}{\sqrt{4\pi}} \sum_{c=1}^3 \{ A'_c \exp(-|\beta'_c \vec{r}_c + \beta_c \vec{R}_c|) + A_c \exp(-|\alpha'_c \vec{r}_c + \alpha_c \vec{R}_c|) \} \frac{\chi_c^{J,v}(R_c)}{R_c} Y_{J,M}(\hat{R}_c), \quad (7)$$

$$H = -\frac{\hbar^2}{2m_c} \nabla_{\vec{r}_c}^2 - \frac{\hbar^2}{2M_c} \nabla_{\vec{R}_c}^2 + V_C(\vec{r}_c, \vec{R}_c). \quad (8)$$

The expansion coefficients are optimized versus the sets of parameters $\{\beta_c^{(j)}\}$ and $\{\alpha_c^{(j)}\}$, by the Rayleigh-Ritz formula. Use of the form of equation (7) for the wavefunction, rather than some functions, makes the transformation between the three sets of the Jacobian coordinates simpler.

The coupled channel method is also applied in the muon-induced fission [17]. Karpeshin used the complex trajectory method and discussed the interplay of different channels in the scattering problem [17]. In the present method alternative to [17] and others [5, 18], some of the components of the three-body wavefunction have the analytical forms, and other components, $\chi_c(R_c)$, are calculated numerically. We make integrations on the muon coordinate in each channel separately, and simplify the Rayleigh–Ritz formula as follows:

$$\langle Y_{JM}(\hat{R}_c) | H - E | \psi_I \rangle_{\vec{r}_c, \hat{R}_c} = 0 \quad \text{and} \quad c = 1, 2 \quad \text{and} \quad 3. \quad (9)$$

Here, $\langle \rangle_{\vec{r}_c, \hat{R}_c}$ denotes the integration over \vec{r}_c and \hat{R}_c . In the three types of Jacobian coordinates, we have

$$\begin{aligned} & -\frac{2\hbar^2}{M_c} \left\{ \frac{A'_c}{\beta'^3_c} + \frac{A_c}{\alpha^3_c} \right\} \left(\frac{d^2}{dR_c^2} - \frac{J(J+1)}{R_c^2} \right) \chi_c^{J,v}(R_c) \\ & - \left\{ A'_c \left(\frac{\hbar^2}{M_c \beta'_c} + \frac{\hbar^2}{m_c \beta'_c} \right) + A_c \left(\frac{\hbar^2}{M_c \alpha_c} + \frac{\hbar^2}{m_c \alpha'_c} \right) - V_{\text{eff}}^c(R_c) \right\} \chi_c^{J,v}(R_c) \\ & = 4\epsilon_{J,v} \left\{ \frac{A'_c}{\beta'^3_c} + \frac{A_c}{\alpha^3_c} \right\} \chi_c^{J,v}(R_c) - \int_0^\infty V^{c,c'}(R_c, R_{c'}) \chi_{c'}^{J,v}(R_{c'}) dR_{c'} \\ & - \int_0^\infty V^{c,c''}(R_c, R_{c''}) \chi_{c''}^{J,v}(R_{c''}) dR_{c''}, \\ & (c, c', c'') = (1, 2, 3), (2, 3, 1) \quad \text{and} \quad (3, 1, 2), \end{aligned} \quad (10)$$

where

$$\begin{aligned} V^{c,c_0}(R_c, R_{c_0}) = & -R_c R_{c_0} \langle Y_{JM}(\hat{R}_c) | Y_{JM}(\hat{R}_{c_0}) \rangle \left\{ -V_{\text{eff}}^{c_0}(R_{c_0}) + A'_{c_0} \left(\frac{\hbar^2}{M_{c_0} \beta'_{c_0}} + \frac{\hbar^2}{m_{c_0} \beta'_{c_0}} \right) \right. \\ & \left. + A_{c_0} \left(\frac{\hbar^2}{M_{c_0} \alpha_{c_0}} + \frac{\hbar^2}{m_{c_0} \alpha'_{c_0}} \right) - 4 \left(\frac{A'_{c_0}}{\beta'^3_{c_0}} + \frac{A_{c_0}}{\alpha^3_{c_0}} \right) \left(\frac{\hbar^2}{2M_{c_0}} \frac{J(J+1)}{R_{c_0}^2} - \epsilon_{J,v} \right) \right\}. \end{aligned} \quad (11)$$

where $V_{\text{eff}}^c(R_c)$ is the effective potential involving linear-exponential integrals,

$$V_{\text{eff}}^c(R_c) = \sum_{i=1}^4 V_{\text{eff}}^{c,i}(R_c) \quad \text{and} \quad c = (1, 2, 3), \quad (12)$$

which are obtained as

$$\begin{aligned} V_{\text{eff}}^{1,1}(R_1) = & \frac{A'_1 e^2}{\beta'_1 \beta_1 R_1} \int_0^\infty \{ e^{-|\beta'_1 r' + \beta_1 R_1|} (|\beta'_1 r' + \beta_1 R_1| + 1) \\ & - e^{-|\beta'_1 r' - \beta_1 R_1|} (|\beta'_1 r' - \beta_1 R_1| + 1) \} dr', \end{aligned} \quad (13)$$

$$V_{\text{eff}}^{1,2}(R_1) = V_{\text{eff}}^{1,1}(R_1; \{A'_1, \beta'_1, \beta_1\} \rightarrow \{A_1, \alpha'_1, \alpha_1\}), \quad (14)$$

$$\begin{aligned} V_{\text{eff}}^{1,3}(R_1) = & \frac{A'_1 e^2}{\delta_1^2 R_1 \beta'_1 (\delta_1 \beta_1 + \beta'_1)} \int_0^\infty \{ e^{-\frac{1}{\delta_1} |\beta'_1 r' + (\delta_1 \beta_1 + \beta'_1) R_1|} (|\beta'_1 r' + (\delta_1 \beta_1 + \beta'_1) R_1| + \delta_1) \\ & - e^{-\frac{1}{\delta_1} |\beta'_1 r' - (\delta_1 \beta_1 + \beta'_1) R_1|} (|\beta'_1 r' - (\delta_1 \beta_1 + \beta'_1) R_1| + \delta_1) \} dr' \\ & - \frac{A'_1 e^2}{\delta_1^2 R_1 \beta'_1 (\delta'_1 \beta_1 - \beta'_1)} \int_0^\infty \{ e^{-\frac{1}{\delta'_1} |\beta'_1 r' + (\delta'_1 \beta_1 - \beta'_1) R_1|} (|\beta'_1 r' + (\delta'_1 \beta_1 - \beta'_1) R_1| + \delta'_1) \\ & - e^{-\frac{1}{\delta'_1} |\beta'_1 r' - (\delta'_1 \beta_1 - \beta'_1) R_1|} (|\beta'_1 r' - (\delta'_1 \beta_1 - \beta'_1) R_1| + \delta'_1) \} dr', \end{aligned} \quad (15)$$

$$\delta_1 = m_d / (m_\mu + m_d) \quad \text{and} \quad \delta'_1 = 1 - \delta_1, \quad (15)$$

$$V_{\text{eff}}^{1,4}(R_1) = V_{\text{eff}}^{1,3}(R_1; \{A'_1, \beta'_1, \beta_1\} \rightarrow \{A_1, \alpha'_1, \alpha_1\}), \quad (16)$$

$$V_{\text{eff}}^2(R_2) = V_{\text{eff}}^1(\{R_1, \delta_1, \delta'_1\} \rightarrow \{R_2, \delta_2, \delta'_2\}), \quad \delta_2 = m_p/(m_\mu + m_p), \quad (17)$$

$$V_{\text{eff}}^{3,1}(R_3) = \frac{A'_3 e^2}{R_3 \beta'_3 (\beta_3 + \delta_3 \beta'_3)} \int_0^\infty \{e^{-|\beta'_3 r' + (\beta_3 + \delta_3 \beta'_3) R_3|} (|\beta'_3 r' + (\beta_3 + \delta_3 \beta'_3) R_3| + 1) - e^{-|\beta'_3 r' - (\beta_3 + \delta_3 \beta'_3) R_3|} (|\beta'_3 r' - (\beta_3 + \delta_3 \beta'_3) R_3| + 1)\} dr', \quad (18)$$

$$\delta_3 = m_p/(m_d + m_p), \quad (18)$$

$$V_{\text{eff}}^{3,2}(R_3) = V_{\text{eff}}^{3,1}(R_3; \{A'_3, \beta'_3, \beta_3\} \rightarrow \{A_3, \alpha'_3, \alpha_3\}), \quad (19)$$

$$V_{\text{eff}}^{3,3}(R_3) = 4 \frac{A'_3 e^2}{\beta'^3_3 R_3} + V_{\text{eff}}^{3,1}(R_3; \delta_3 \rightarrow \delta_3 - 1), \quad (20)$$

$$V_{\text{eff}}^{3,4}(R_3) = 4 \frac{A_3 e^2}{\alpha'^3_3 R_3} + V_{\text{eff}}^{3,2}(R_3; \delta_3 \rightarrow \delta_3 - 1).$$

The energy E and the radial wavefunctions are obtained versus the ro-vibrational quantum numbers, $E \rightarrow \epsilon_{J,v}$. The optimal values for the molecular parameters are given in table 1. The obtained value of $\epsilon_{0,0}$ is -221.568 eV, close to the previously reported [10, 19]. The pervious energy value of the ground state of $pd\mu$ was -221.55 eV. The channel $c = 3$ is the most important to determine the fusion rate in the nuclear fusion region. For this purpose, we obtain the probability density versus the nuclear distance by the integration over the muon coordinate in the channel $c = 3$ as follows:

$$\rho_{r_{pd}} = \int |\psi_I(\vec{r}_\mu, \vec{r}_{pd})|^2 d\vec{r}_\mu d\Omega_{\vec{r}_{pd}}. \quad (21)$$

In our calculations, since $A_3 \cong A'_3$ and $\beta'_3 = \alpha'_3$, the probability density equals

$$\rho_0 \cong \lim_{R_3 \rightarrow 0} |\chi_3^{J=0, v=0}(R_3)/R_3|^2, \quad (22)$$

for the small distances. The adiabatic picture was also used for the classification of the molecular states $\epsilon_{J,v}$ in the previous methods [10, 19]. In particular, the local characteristic such as the probability density (6) was calculated using the 15 terms of the adiabatic hyperspherical expansion [10, 19]. In this method [10, 19] and also in the Faddeev equations [1–3], the applied wavefunctions and the potential terms have different forms than those of our work. The theoretical description of the $p + d \rightarrow {}^3\text{He} + \gamma$, at low energy physics, is complicated by the presence the Coulomb interaction. Only relatively recently has the s-wave capture contribution to the zero energy astrophysical factor of this reaction. It has been calculated with the numerically converged Faddeev wavefunctions [15]. Friar *et al* calculated this nuclear factor using equation (5). The calculated values for $S(0)$ and the ratio $\sqrt{\frac{S_{3/2}(0)}{S_{1/2}(0)}}$ have been found to be $0.108(4) \text{ eV} - b$ and $0.54(2)$ respectively [15], in excellent agreement with the recent experimental determinations, $S(0) = 0.109(10) \text{ eV} - b$ and $\sqrt{\frac{S_{3/2}(0)}{S_{1/2}(0)}} = 0.50(15)$ [20]. For more data, Karpeshin *et al* derived an expression for the internal conversion (reaction (1)) in the non-relativistic limit by the use of the Dirac wavefunctions [21]. The same procedure can be repeated for the radiative width in the case of the $M1$ transition, as the latter expression differs from the first one by replacement of the spherical Bessel function instead of the Hankel one [22]. We substitute the experimental values of nuclear factors of [20] and ρ_0 of equation (22) into equation (3) to estimate the radiative fusion rates. The calculated radiative fusion rates are compared with both the experimental [9] and the previous theoretical rates [15], as given in table 2.

Table 1. Optimal values of the $pd\mu$ parameters. $A_c^{(i)}$ is given in the unit of $(10^{15} \text{ cm}^{-3/2})$, the parameter sets $\{\beta_c^{(i)}\}$ and $\{\alpha_c^{(i)}\}$ are in the unit of $(10^{10} \text{ cm}^{-1})$.

Parameter	Value
β'_1	3.420
β_1	-0.001
α'_1	3.752
α_1	0.002
β'_2	3.501
β_2	-0.003
α'_2	3.834
α_2	0.002
β'_3	4.000
β_3	-2.434
α'_3	4.001
α_3	5.009
A'_1	12.608
A_1	0.032
A'_2	13.090
A_2	0.019
A'_3	7.999
A_3	7.991

Table 2. The calculated radiative fusion rates (μs^{-1}) and their comparison with both the experimental [9] previous theoretical rates [15].

Calculated rate $\lambda_{3/2}^\gamma$	Previous theoretical rate [15]	Experimental value [9]
0.08	0.107(6)	0.11(1)
Calculated rate $\lambda_{1/2}^\gamma$	Previous theoretical rate	Experimental value
0.32	0.37(1)	0.35(2)

3. Conclusions

The results and view of this paper can be summarized as follows. The main idea of our computation technique is the introduction of the new and simple wavefunction of equation (7). First, the three coupled integro-differential equations for $\chi_1^{J,v}(R_1)$, $\chi_2^{J,v}(R_2)$ and $\chi_3^{J,v}(R_3)$, equation (10), are solved using a numerical procedure of Rung–Kutta 45. Then, the integration of $\rho_{r_{pd}}$ is simplified to $|\chi_3^{J=0,v=0}(R_3)/R_3|^2$ at $R_3 \rightarrow 0$, by hand. The product of ρ_0 and $\frac{S_{3/2}(0)}{\pi\alpha cm_3}$ ($\frac{S_{1/2}(0)}{\pi\alpha cm_3}$) makes the quartet (doublet) rate value. As the present approach requires comparatively six terms for the discussion on the molecular wavefunction, the mathematical work is easily done. The obtained results are close to those previously reported, see table 2. The validity of our approximation is only for $R \approx 0$ to 1.9×10^{-8} cm. The main limitations of this method for the problem under discussion are given as follows. The adiabatic approximation is used in our work, i.e. we make integrations on the muon coordinates in each channel separately. These include some physics limits. The wavefunction of equation (7) is employed to obtain some molecular parameters, as averaged on the muon coordinates. In this method, the overlap of the wavefunctions of the three channels is not considered. Although, the above mentioned method is a powerful one. Kamimura used the three-channel method employing a molecular

wavefunction [18], alternative to the present wavefunction. The results of his work were very exact [18]. The accuracy of the present work decreases for larger J . For $J \geq 2$, poor convergence occurs in the curve of $\chi_c(R_c)$. The corrections to the results of the non-relativistic equation (10) such as the finite nuclear size, vacuum polarization, molecular environment, relativistic terms become important for low states of $pd\mu$. The present wavefunction is the base data. The proposed variational method will be computationally convenient with respect to some of the three-body calculations in further studies. Extensive calculations of collisions of muonic atoms in the 1s state with hydrogen atoms (or molecules), the muon transfer in the atomic cascade and the muon sticking in the $pd\mu$ cycle can be done with this variational molecular wavefunction. The channel $c = 3$ is more suitable for the study of muon sticking. The channels $c = 1$ and 2 are the most important for scattering of muonic atoms and muon transfer. These calculations can be repeated for mass-nonsymmetric molecules such as $pt\mu$. Only the $pd\mu$ and $pt\mu$ molecules have important roles in the Ramsauer–Townsend (RT) effect [10].

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References

- [1] Kostrykin A A, Kvitsinsky A A and Merkuriev S P 1989 *Few-Body Syst.* **6** 97
- [2] Hu C, Kvitsinsky A A and Merkuriev S P 1992 *Phys. Rev. A* **45** 2723
- [3] Hu C and Kvitsinsky A A 1992 *Phys. Rev. A* **46** 7301
- [4] Bogdanova L N, Markushin V E, Melezhik V S and Ponomarev L I 1981 *Sov. J. Nucl. Phys.* **34** 662
- [5] Kamimura M 1989 *Muon-Catalyzed Fusion (AIP Conf. Proc. vol 181)* ed S E Jones, J Rafelski and H J Monkhorst (New York: AIP) pp 330–43
- [6] Szalewicz K *et al* 1990 *Phys. Rev. A* **42** 3768
- [7] Hu C Y, Hale G M and Cohen J S 1994 *Phys. Rev. A* **49** 4481
- [8] Bogdanova L N and Markushin V E 1990 *Muon Catal. Fusion* **5/6** 189
- [9] Petitjean C *et al* 1990/91 *Muon Catal. Fusion* **5/6** 199
- [10] Markushin V E 1999 *Few-Body Syst. Suppl.* **10** 169
- [11] Adamczak A 1996 *Hyperfine Interact.* **101/102** 113
- [12] Alvarez L W *et al* 1957 *Phys. Rev.* **105** 1127
- [13] Bossy H *et al* 1985 *Phys. Rev. Lett.* **55** 1870
- [14] Gerstein S S 1961 *Sov. Phys.—JETP* **13** 488
- [15] Friar J L, Gibson B F, Jean H C and Payne G L 1991 *Phys. Rev. Lett.* **66** 1827
- [16] Cohn S, Judd D L and Riddell R J 1960 *Phys. Rev.* **119** 384
- [17] Karpeshin F F 2004 *J. Phys. G: Nucl. Part. Phys.* **30** 1
- [18] Kamimura M 1988 *Phys. Rev. A* **38** 621
- [19] Abramov D I *et al* 1998 *Phys. At. Nucl.* **61** 457
- [20] Schmid G J *et al* 1996 *Phys. Rev. Lett.* **76** 3088
- [21] Karpeshin F F *et al* 1996 *Phys. Rev. C* **53** 1640
- [22] Viviani M, Kievsky A and Schiavilla R 1996 *Phys. Rev. C* **54** 534